Zeidan et al., 2016
J. basic appl. Res 2(4): 495-502

JBAAR
Journal of Basic And Applied Research
ISSN 2413-7014

Statistical Interval for Data Envelopment Analysis

Asmaa S. A. Zeidan[1], Enayat I. Hafez [2], Elham Abd El-Raziq[3]

[1]Assistant Lecturer, Department of Statistics, Faculty of Commerce, Al-Azhar University (Girls' Branch), Cairo, Egypt.
[2] Professor of Operations Research, Department of Statistics, Faculty of Commerce, Al-Azhar University (Girls' Branch), Cairo, Egypt.
[3] Professor of Operations Research, Department of Statistics, Faculty of Commerce, Al-Azhar University (Girls' Branch), Cairo, Egypt.

*corresponding author: asmaanile2020@yahoo.com

Received: 3-6-2016
Revised: 18-6-2016
Published: 6-7-2016

Keywords:
Data envelopment analysis; fuzzy; Interval data; Efficiency; Statistical confidence; Decision making units.

Abstract: The techniques of data envelopment analysis (DEA) were largely studied. Data envelopment analysis (DEA) is a methodology for measuring the relative efficiencies of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs. Conventional DEA models assume that input and output values should be certain (crisp data). However, the observed values of the input and output data in real-world situations are sometimes inexact, incomplete, vague, ambiguous or imprecise. Some researchers have proposed various methods for dealing with the imprecise and ambiguous data in DEA in the context of fuzzy (interval) data. In this paper, a statistical method based on arithmetic operations to solve fuzzy (interval) data envelopment analysis models (FDEA) can be improved. The suggested approach transforms the original data (crisp data) into interval data; in the form of upper and lower frontier data. Then, by using these upper and lower frontier data; the interval DEA efficiency scores can be achieved. This approach is applied on the real-life data and the results of application are efficient.

Cite this article as: Zeidan, A.S.A., Hafez, E.I., Abd El-Raziq, E. (2016). Statistical Interval for Data Envelopment Analysis. Journal of basic and applied Research 2(4): 495-502

INTRODUCTION

Data envelopment analysis (DEA) is a non-parametric technique for evaluating and measuring the relative efficiency of decision making units (DMUs) characterized by multiple inputs and multiple outputs. DEA is a linear programming technique that computes a comparative ratio of weighted outputs to weighted inputs for each unit, which is reported as the relative efficiency score. The efficiency score is usually expressed as either a number between zero and one (0-1) or as a percentage (0-100%). A decision-making unit with a score equal one becomes the efficient unit. On the other hand, a unit with a score less than one is deemed inefficient relative to other efficient units (Avkiran, 2001, Jablonsky, 2013).

The name of DEA was due to constructing an efficient frontier from efficient units by the model that this frontier will cover (envelope) the inefficient units (Kazemi and Alimi, 2014).

DEA has initially been used to investigate the relative efficiency of non-for-profit organizations and it is quickly spread to profit-making organizations. DEA has been successfully applied in such diverse settings as schools, universities, hospitals, libraries, banks, shops, industries, and more recently, whole economic and society systems; in which outputs and inputs are always multiple (Iddrisu, 2014, Abd-Aziz et al., 2013).

DEA is based on the study of Farrell in 1957. Farrell's seminal work was the first to propose the concept of technical efficiency, stating that technical efficiency is the ability of a firm to obtain maximal output for a given set of inputs. Farrell's definition of technical efficiency led to the development of methods for estimating relative efficiencies of multi-input multi-output production units (Mohammadi and Ranaei, 2011).

Twenty years after Farrell's seminal work 1957, and as responding to the need for satisfactory procedures to assess the relative efficiencies of multi-input multi-output production units, Charnes et al. (1978) put into practice Farrell's view for the first time and introduced a powerful methodology as an evaluation tool to measure the relative efficiencies of decision making units (DMUs) and named it as "Data Envelopment Analysis". Their DEA approach applies linear programming techniques to observed inputs consumed and output produced by decision-making units and constructs an efficient production frontier based on the best practices. Each DMU's efficiency is then measured relative to this frontier.

Since the advent of DEA seminal paper in 1978, a large literature on DEA has developed, focusing both on methodological (theoretical) developments and practical applications. Moreover; all other models are extensions of that approach.

This paper is organized as follows. The next section contains conventional models of DEA. Section 3 presents a discussion about FDEA including fuzzy set theory, fuzzy numbers, fuzzy DEA models, interval DEA, and the suggested method to express the data as interval data. In section 4, an application based on a real data is presented. Section 5 closes with final results and conclusion.

I. CONVENTIONAL MODELS OF DATA ENVELOPMENT ANALYSIS

The models of data envelopment analysis were studied by many authors. Over the last decades, the field of usage of DEA models has been extensively updated. The basic idea for development of DEA models is to enable the efficiency measurement in non-profit sector where there are no exact financial measures. Later, DEA models were applied also in the profit sector. Numerous applications have caused the development of new methods and models, but in this section, for the purpose of understanding the basics of DEA, only CCR (Charnes, Cooper and Rhodes) BCC (Banker, Charns and Cooper) models are presented.

A. The CCR Model

The DEA model originally proposed by Charnes, Cooper, and Rhodes is called the CCR model (which is named after the first letters of their names). First, their proposed measure of the efficiency of any DMU is obtained as the maximum of a ratio of weighted outputs to weighted inputs subject to the condition that; the similar ratios for every DMU be less than or equal to unity.

They assumed that there are n of DMU_S to be evaluated, where every DMU_j $(j = 1,2,\dots\dots,n)$ consumes varying amounts of m different inputs $x_{ij}(i = 1,2,\dots\dots,m)$ to produce s different output$y_{rj}(r = 1,2,\dots\dots,s)$. With decision variables outputs weights $u_r(r = 1,2,\dots,s)$ and inputs weights v_i $(i = 1,2,\dots,m)$ being selected, the mathematical formulation of the method is summarized as follows:

$$max \qquad h_0 = \frac{\sum_{r=1}^{s} u_r\ y_{r0}}{\sum_{i=1}^{m} v_i\ x_{i0}}$$

$$Subject\ to: \ \frac{\sum_{r=1}^{s} u_r\ y_{rj}}{\sum_{i=1}^{m} v_i\ x_{ij}} \leq 1 \ ;\ j = 1,2,\dots\dots,n$$

$$u_r, v_i \geq 0 \quad ; \quad r = 1,\dots,s\ ; \quad i = 1,\dots,m$$

$$\Rightarrow \quad (1)$$

Hence, the fractional CCR model (1) evaluates the relative efficiencies of n decision making units (DMUs), each of them with m inputs and s outputs by maximizing the ratio of h_0.

The fractional programming model (1) can be transformed into a linear form as follows:

$$max \qquad h_0 = \sum_{r=1}^{s} u_r\ y_{r0}$$

$$Subject\ to: \quad \sum_{i=1}^{m} v_i\ x_{i0} = 1$$

$$\sum_{r=1}^{s} u_r\ y_{rj} - \sum_{i=1}^{m} v_i\ x_{ij} \leq 0 \ ; \qquad j = 1,\dots,n$$

$$u_r, v_i \geq 0 \ ; \quad r = 1,\dots,s\ ; \ i = 1,\dots,m$$

$$\Rightarrow \quad (2)$$

B. The BCC Model

Banker, Charnes, and Cooper 1984 introduced the BCC model (which is named after the first letters of their names). This model is an extension of the CCR model. The primary difference between the two models (CCR and BCC) is the treatment of returns to scale. Charnes, Cooper, and Rhodes assumed constant returns to scale (CRS) that means; an increment (a rise) in inputs results in proportion increment in outputs. On the other hand, Banker, Charnes, and Cooper assumed variable returns to scale (VRS) which means; an increment in inputs results in a disproportionate increment in outputs. So, the BCC model is more flexible. These two radial models can be easily illustrated in the following two figures (Fig. 1 and fig. 2) (Tlig, 2013)

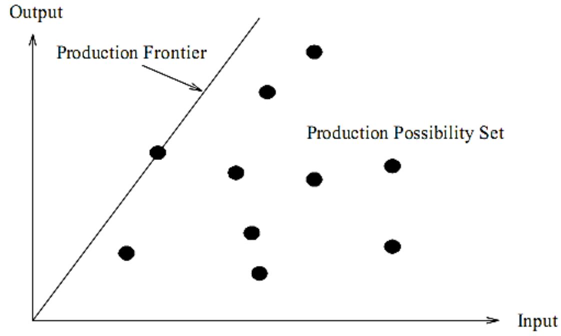

Fig. 1: Production frontier of the CCR model

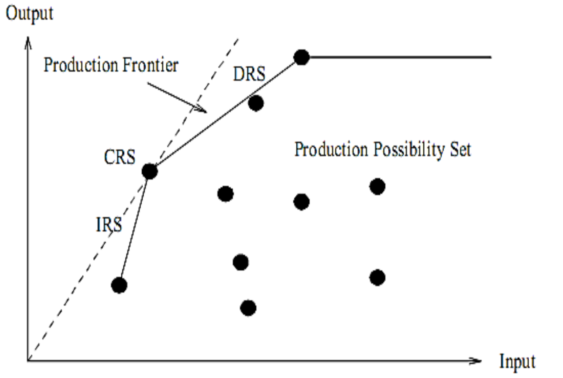

Fig. 2: Production frontier of the BCC model

The BCC ratio model differs from the CCR ratio model (1), by an additional variable as follows:

$$max \qquad h_0 = \frac{\sum_{r=1}^{s} u_r\ y_{r0} - c_0}{\sum_{i=1}^{m} v_i\ x_{i0}}$$

$$Subject\ to: \frac{\sum_{r=1}^{s} u_r\ y_{rj} - c_0}{\sum_{i=1}^{m} v_i\ x_{ij}} \leq 1\ ; j = 1,\dots,n$$

$$u_r, v_i \geq 0\ ; \quad r = 1,2,\dots\dots,s\ ; \quad i = 1,2,\dots\dots,m$$

$$c_0 \quad unrestricted\ in\ sign$$

$$\Rightarrow \quad (3)$$

Where c_0 is the new variable that separates scale efficiency from technical efficiency in the CCR model.

The BCC primal linear programming model that measures pure technical efficiency is given as follows:

$$max \qquad h_0 = \sum_{r=1}^{s} u_r\ y_{r0} - c_0$$
$$Subject\ to: \quad \sum_{i=1}^{m} v_i\ x_{i0} = 1$$
$$\sum_{r=1}^{s} u_r\ y_{rj} - \sum_{i=1}^{m} v_i\ x_{ij} - c_0 \le 0\ ; \quad j = 1, \dots, n$$
$$u_r, v_i \ge 0 \quad ; \qquad r = 1, \dots, s\ ; \quad i = 1, \dots, m$$
$$c_0 \quad unrestricted\ in\ sign$$
$$\Rightarrow \qquad (4)$$

When ($c_0 = 0$), it implies CRS (constant returns to scale). If ($c_0 > 0$), it implies DRS (decreasing returns to scale), and if ($c_0 < 0$), it implies IRS (increasing returns to scale) (Argyrioy and Sifaleras, 2013) & (Avkiran, 2001).

II. FUZZY DATA ENVELOPMENT ANALYSIS

The traditional data envelopment analysis (DEA) models use crisp values and precise input and output data to evaluate efficiencies. But, in point of fact, it is not always possible to work with certain values due to various reasons. One of them is that; in real-world problems, the observed values of the input and output data are sometimes imprecise or vague. Imprecise or vague data may be the result of unquantifiable, incomplete and unobtainable information. To deal with imprecise data, fuzzy set theory has become an effective method to quantify imprecise and vague data in DEA models. Another reason; in many situations, such as in a manufacturing system, a production process or a service system, inputs and outputs are volatile and complex so that it is difficult to measure them in an accurate way. Instead the data can be given as in forms of bounded or fuzzy data. Furthermore, the data can be represented by linguistic terms, e.g. good, medium, or bad. In these cases, fuzzy set theory can be a powerful tool to deal with the linguistic variables. So, many researchers have proposed various fuzzy methods for dealing with the imprecise and ambiguous data in DEA (Isabels and Uthra, 2012).

A. Fuzzy set theory

Zadeh (1965) was the first one who introduced the concept of fuzzy sets. According to Zadeh, a fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Fuzzy set algebra developed by Zadeh is the formal body of the theory of fuzzy sets that allows the treatment of imprecise and vague data in uncertain environments.

Fuzzy set theory is a generalization of classical set theory in that the domain of the characteristics function is extended from the discrete set $\{0, 1\}$ to the closed real interval $[0, 1]$. Zadeh (1965) defined a fuzzy set as a class of objects with continuum grades of membership.

Mansourirad et al. (2010) showed that; X is a space of objects and x is a generic element of X. A fuzzy set, $\tilde{A}$, in X can be defined as:

$$\tilde{A} = \{(x, \mu_A(x)) \mid x \in X\} \qquad (5)$$

Where $\mu_A(x): X \to M$ is the membership function and M is the membership space that varies in the interval $[0, 1]$. The closer the value of $\mu_A(x)$ is to one, the greater the membership degree of X to $\tilde{A}$. However, if $M = \{0, 1\}$, the set A is non-fuzzy. A fuzzy set $\tilde{A}$ can be defined precisely by associating with each object x a number between 0 and 1, which represents its grade of membership in A. Thus, $\mu_A(x)$ = 1 if x is totally in A, $\mu_A(x) = 0$ if x is not in A, and $0 < \mu_A(x) < 1$if x is partly in A.

Recently, Fuzzy set theory has been applied to a wide range of fields such as management science, decision theory, artificial intelligence, computer science, expert systems, logic, control theory and statistics.

B. Fuzzy numbers

A fuzzy number is an extension of a regular number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function (Mansourirad et al., 2010). There are many different types of fuzzy numbers; our attention will be focused on interval fuzzy numbers as it will be used in forming the fuzzy linear programming models.

C. Fuzzy Data Envelopment Analysis models

Sengupta (1992) was the first to introduce a fuzzy mathematical programming approach in which fuzziness was incorporated into DEA by allowing both the objective function and the constraints to be fuzzy. The author explored the use of fuzzy set theory in decision making. In the study, three types of fuzzy models (fuzzy mathematical programming, fuzzy regression and fuzzy entropy) were presented to illustrate the types of decisions and solutions that were achievable, when the data are vague and prior information is inexact and imprecise.

i. The Fuzzy CCR Model

Assume that there are n of DMU_S to be evaluated, where every DMU_j $(j = 1,2, \dots \dots, n)$ consumes varying amounts of m different inputs $\tilde{x}_{ij} (i = 1,2, \dots \dots, m)$ to produce s different outputs $\tilde{y}_{rj} (r = 1,2, \dots \dots, s)$. Where ($\tilde{x}_{ij}$, $\tilde{y}_{rj}$) represent, respectively, the fuzzy input and fuzzy output of the jth DMU_j $(j = 1,2, \dots \dots, n)$ With decision variables outputs weights u_r $(r = 1,2, \dots \dots, s)$ and inputs weights v_i $(i = 1,2, \dots \dots, m)$ being selected, the fractional CCR model with fuzzy data can be formulated as follows:

$$max \qquad h_0 = \frac{\sum_{r=1}^{s} u_r\ \tilde{y}_{r0}}{\sum_{i=1}^{m} v_i\ \tilde{x}_{i0}}$$

$$Subject\ to: \frac{\sum_{r=1}^{s} u_r\ \tilde{y}_{rj}}{\sum_{i=1}^{m} v_i\ \tilde{x}_{ij}} \le 1 \quad ; \quad j = 1, \dots, n$$
$$u_r, v_i \ge 0 \quad ; \ r = 1, \dots, s \ ; \quad i = 1,2, \dots \dots, m \qquad \Rightarrow (6)$$

Where "~" indicate the fuzziness.

The fuzzy fractional programming model (6) can be transformed into fuzzy linear programming model. The CCR model with fuzzy data (coefficients) can be written as:

$$max \quad h_0 = \sum_{r=1}^{s} u_r\ \tilde{y}_{r0}$$
$$Subject\ to: \quad \sum_{i=1}^{m} v_i\ \tilde{x}_{i0} = 1$$
$$\sum_{r=1}^{s} u_r\ \tilde{y}_{rj} - \sum_{i=1}^{m} v_i\ \tilde{x}_{ij} \le 0 \ ; \quad j = 1, \dots, n$$
$$u_r, v_i \ge 0 \quad ; \quad r = 1, \dots, s\ ; \ i = 1, \dots, m \qquad \Rightarrow (7)$$

ii. The Fuzzy BCC Model

By the same way, the BCC linear programming model with fuzzy data is given as follows:

$$max \quad h_0 = \sum_{r=1}^{s} u_r\ \tilde{y}_{r0} - c_0$$
$$Subject\ to: \quad \sum_{i=1}^{m} v_i\ \tilde{x}_{i0} = 1$$
$$\sum_{r=1}^{s} u_r\ \tilde{y}_{rj} - \sum_{i=1}^{m} v_i\ \tilde{x}_{ij} - c_0 \ \le 0 \ ; \quad j = 1, \dots, n$$
$$u_r, v_i \ge 0 \quad ; \quad r = 1, \dots, s\ ; \ i = 1, \dots, m \qquad \Rightarrow (8)$$

Where "~" indicate the fuzziness.

The interpretation of constraints of FCCR and FBCC models is similar to the crisp CCR and BCC models. The difference between the two models resides on the manner of resolution. The crisp CCR model can be simply solved by a standard LP solver. For the FCCR model, the resolution is more difficult and requires methods for fuzzy sets (Tlig, 2013).

iii. The Interval DEA

As mentioned before, there are many different types of fuzzy numbers; our attention will be focused on interval fuzzy numbers. In a condition that all inputs and outputs are not totally available due to uncertainties, these values are only known to lie within the upper and lower bounds represented by intervals $[x_{ij}{}^{L},\ x_{ij}{}^{U}]$ and $[y_{rj}{}^{L}, y_{rj}{}^{U}]$, where $x_{ij}{}^{L} > 0$ and $y_{rj}{}^{L} > 0$. In order to deal with such an uncertain situation, the following pair of linear fractional models has been developed to generate the upper and lower bounds of interval efficiency for each DMU. Therefore, model (6) can be re-written as follows: (Wang et. al, 2005)

$$max \qquad h_0^U = \frac{\sum_{r=1}^{s} u_r\ y^{U}{}_{r0}}{\sum_{i=1}^{m} v_i\ x^{L}{}_{i0}}$$
$$Subject\ to: \quad \frac{\sum_{r=1}^{s} u_r\ y^{U}{}_{rj}}{\sum_{i=1}^{m} v_i\ x^{L}{}_{ij}} \le 1\ ; \ j = 1, \dots, n$$
$$u_r, v_i \ge 0 \quad ; \ r = 1,2, \dots \dots, s\ ; \ i = 1,2, \dots \dots, m \qquad \Rightarrow (9)$$

$$max \qquad h_0^L = \frac{\sum_{r=1}^{s} u_r\ y^{L}{}_{r0}}{\sum_{i=1}^{m} v_i\ x^{U}{}_{i0}}$$
$$Subject\ to: \quad \frac{\sum_{r=1}^{s} u_r\ y^{U}{}_{rj}}{\sum_{i=1}^{m} v_i\ x^{L}{}_{ij}} \le 1\ ; \quad j = 1, \dots, n$$
$$u_r, v_i \ge 0 \quad ; \quad r = 1, \dots, s\ ; \quad i = 1, \dots, m \qquad \Rightarrow (10)$$

The fractional programming models (9) and (10) can be transformed into linear programming models as follows:

$$max \quad h_0^U = \sum_{r=1}^{s} u_r\ y^{U}{}_{r0}$$
$$Subject\ to: \quad \sum_{i=1}^{m} v_i\ x^{L}{}_{i0} = 1$$
$$\sum_{r=1}^{s} u_r\ y^{U}{}_{rj} - \sum_{i=1}^{m} v_i\ x^{L}{}_{ij} \le 0 \ ; \quad j = 1, \dots, n$$
$$u_r, v_i \ge 0 \quad ; \quad r = 1, \dots, s\ ; \ i = 1, \dots, m \qquad \Rightarrow (11)$$

$$max \quad h_0^L = \sum_{r=1}^{s} u_r\ y^{L}{}_{r0}$$
$$Subject\ to: \quad \sum_{i=1}^{m} v_i\ x^{U}{}_{i0} = 1$$
$$\sum_{r=1}^{s} u_r\ y^{U}{}_{rj} - \sum_{i=1}^{m} v_i\ x^{L}{}_{ij} \le 0 \ ; \quad j = 1, \dots, n$$
$$u_r, v_i \ge 0 \quad ; \quad r = 1, \dots, s\ ; \quad i = 1, \dots, m \qquad \Rightarrow (12)$$

Where h_0^U stands for the upper bound of the best possible relative efficiency of DMU_0, and h_0^L stands for the lower bound of the best possible relative efficiency of DMU_0.

Demir (2014) suggested solving the two models (11) and (12) by changing the crisp data into interval data. Upper and lower frontier data were calculated by adding and removing standard errors to each variable, and so each data was turned into interval data. To calculate upper frontier efficacy scores, upper frontier values of the output data and lower frontier values of the input data were used. When it came to the lower frontier efficacy scores, lower frontier values of the output data and upper frontier values of the input data were used. The formulas are:

(Upper frontier data) = (Available data) + (Standard Error)

(Lower frontier data) = (Available data) - (Standard Error)

$\Rightarrow$ (13)

iv. The suggested Method

In this study, a statistical interval is suggested to express of crisp data as interval data in the form of lower and upper bounds as follows:

Lower bound data =original data - (Standard Error) $*Z\alpha/_2$

Upper bound data =original data + (Standard Error) $*Z\alpha/_2$

$\Rightarrow$ (14)

Although Demir (2014) suggested a method to change the crisp data into interval data by using standard errors of the variables to define the data as interval as mentioned before, the statistical argument for using this method has not been showed. So, the method of Demir is being improved as shown in (14) on the basis of idea of making a statistical confidence interval.

To apply the suggested confidence interval; the data should be distributed as a normal distribution. In other words, this technique assumes that the variables are normally distributed. If a measurement variable does not fit a normal distribution, data transformations should be made. Data transformations such as square root, log, and inverse are commonly used tools that can serve many functions in quantitative analysis of data for improving the normality of variables.

III. APPLICATION AND RESULTS

In order to evaluate the relative efficiency values by using classical and interval DEA models, a real data set of 25high schools in the 2012-2013 education year. The data is taken from Demir (2014). For the purpose of efficiency measurement, numbers of the students, teachers and classes were described as inputs, and Transition to Higher Education Examination (YGS), Undergraduate Placement Exam (LYS) success (placement) rates, YGS point averages, all points of the LYS Maths -Science (MS) , Turkish -Maths (TM), and Turkish-Social (TS) Sciences were described as outputs (see Appendix A).

To evaluate the relative efficiency values by using classical and interval DEA models, several steps are made as follows:

- Calculating the efficiency values of classical DEA models (CCR / BCC).
- Calculating the efficiency values of interval DEA models (CCR / BCC) based on the formulas (13) proposed by Demir.
- Testing whether measurement variables fit a normal distribution or not. If not, data transformations should be made as mentioned before.
- Applying the suggested statistical interval (14) based on three confidence intervals; 90%, 95%, and 99%.

For solving data envelopment analysis (DEA) models, MaxDEA package has been employed.

Efficiency values of classical DEA models (CCR / BCC) are calculated as shown in table (1).

Table (1): Calculated efficiency values with classical DEA models (CCR / BCC)

DMU	Efficiency scores with CCR model	Efficiency scores with BCC model
S1	0.9701	1
S2	0.4515	1
S3	1	1
S4	0.4737	0.5106
S5	0.3693	0.4067
S6	0.5488	0.7185
S7	0.5404	0.5732
S8	1	1
S9	0.8642	1
S10	0.6864	1
S11	1	1
S12	0.2179	0.2221
S13	0.1259	0.1272
S14	0.2196	0.2203
S15	0.2727	0.2742
S16	0.2097	0.2143
S17	1	1
S18	0.9395	1
S19	0.3920	0.3925
S20	0.4203	0.4483
S21	0.2379	0.2392
S22	0.2780	0.2851
S23	0.6759	0.8265
S24	0.3620	0.3815
S25	0.3865	0.3934

In table (1), according to CCR model results; only four units are efficient and the rest of the units are deemed inefficient relative to other efficient units. While the BCC model is more flexible and allows more units to be efficient. So, nine units are efficient and the rest of the units are deemed inefficient relative to other efficient units.

Efficiency values of interval DEA models (CCR / BCC) based on the formulas (13) are calculated and placed on table (2) for lower frontier efficiency and also placed on table (3) for upper bound frontier efficiency as follows:

Table (2): Lower frontier efficiency scores with DEA models (CCR / BCC)

DMU	Lower efficiency values with CCR model	Lower efficiency values with BCC model
S1	1	1
S2	0.5658	1
S3	1	1
S4	0.5527	0.5538
S5	0.4473	0.4635
S6	0.6709	0.7655
S7	0.6336	0.6458
S8	1	1
S9	0.9774	1
S10	0.7709	1
S11	1	1
S12	0.2751	0.2754
S13	0.1588	0.1615
S14	0.2721	0.2737
S15	0.3587	0.3598
S16	0.2605	0.2654
S17	1	1
S18	0.9754	1
S19	0.4905	0.4931
S20	0.5403	0.5481
S21	0.2922	0.2928
S22	0.3423	0.3464
S23	0.7713	0.8672
S24	0.4358	0.4451
S25	0.4352	0.4583

In table (2), according to CCR model results; only five units are efficient and the rest of the units are deemed inefficient relative to other efficient units. While in the BCC model, nine units are efficient and the rest of the units are deemed inefficient relative to other efficient units.

In table (3), according to CCR model results; only three units are efficient and the rest of the units are deemed inefficient relative to other efficient units. While in the BCC model, the results are similar to table (2), so nine units are efficient and the rest of the units are deemed inefficient relative to other efficient units.

To apply the suggested statistical interval (14); the data should be distributed as a normal distribution as mentioned before. This assumption was examined by SPSS program by using Kolmogorov-Smirnov test and it is found that all variables are normally distributed except the variable of LYS- scores MS. This variable can be dealt with by log transformation.

Table (3): Upper frontier efficiency scores with DEA models (CCR / BCC)

DMU	Upper efficiency values with CCR model	Upper efficiency values with BCC model
S1	0.7238	1
S2	0.3199	1
S3	1	1
S4	0.3446	0.458
S5	0.2741	0.3369
S6	0.4259	0.6481
S7	0.4359	0.4727
S8	0.7647	1
S9	0.5831	1
S10	0.4943	1
S11	1	1
S12	0.1637	0.1641
S13	0.0899	0.09
S14	0.156	0.1608
S15	0.1958	0.2073
S16	0.1556	0.1558
S17	1	1
S18	0.8312	1
S19	0.3009	0.3025
S20	0.2512	0.2935
S21	0.1717	0.178
S22	0.2071	0.2111
S23	0.5241	0.7498
S24	0.2832	0.3033
S25	0.3004	0.3108

The previous formulas (14) were calculated based on three of confidence intervals, and so each data was turned into interval data.

Considering 90% confidence interval, efficiency values of interval DEA models (CCR / BCC) are calculated and placed on table (4) for lower frontier efficiency and table (5) for upper frontier efficiency as follows:

Table (4) : Lower frontier efficiency scores with DEA models (CCR / BCC)

DMU	Lower efficiency values with CCR model	Lower efficiency values with BCC model
S1	1	1
S2	0.6301	1
S3	1	1
S4	0.5747	0.5781
S5	0.4926	0.4951
S6	0.7358	0.7884
S7	0.6788	0.681
S8	1	1
S9	1	1
S10	0.8113	1
S11	1	1
S12	0.3088	0.3106
S13	0.1767	0.1824
S14	0.3076	0.3148
S15	0.4012	0.4054
S16	0.2975	0.2976
S17	1	1
S18	0.9884	1
S19	0.5381	0.5448
S20	0.5954	0.5956
S21	0.3196	0.3242
S22	0.3888	0.3916
S23	0.8102	0.8848
S24	0.4789	0.4799
S25	0.4595	0.4935

In table (4), according to CCR model results; the number of efficient units has increased compared with the results of Demir in table (2) and the unit 9 became efficient. While in the BCC model, the results are similar to table (2) so, nine units are efficient and the rest of the units are deemed inefficient relative to other efficient units.

Table (5) : Upper frontier efficiency scores with DEA models (CCR / BCC)

DMU	Upper efficiency values with CCR model	Upper efficiency values with BCC model
S1	0.5359	1
S2	0.2271	1
S3	0.9291	1
S4	0.2501	0.4173
S5	0.1984	0.2858
S6	0.3213	0.5799
S7	0.3374	0.3819
S8	0.5876	1
S9	0.3959	1
S10	0.3574	1
S11	1	1
S12	0.118	0.1238
S13	0.0632	0.0643
S14	0.1106	0.1178
S15	0.14	0.1615
S16	0.1134	0.1168
S17	1	1
S18	0.4958	1
S19	0.2289	0.2327
S20	0.1846	0.1889
S21	0.1249	0.1327
S22	0.1523	0.1543
S23	0.4268	0.649
S24	0.2096	0.244
S25	0.2191	0.2439

In table (5), according to CCR model results; it is found that the number of efficient units has decreased compared with the results of Demir in table (3) and the unit 3 turned to inefficient unit. While in the BCC model, the results are similar to table (3) so, nine units are efficient and the rest of the units are deemed inefficient relative to other efficient units.

Considering 95% confidence interval, efficiency values of interval DEA models (CCR / BCC) are calculated and placed on table (6) for lower frontier efficiency and table (7) for upper frontier efficiency as follows:

Table (6) : Lower frontier efficiency scores with DEA models (CCR / BCC)

DMU	Lower efficiency values with CCR model	Lower efficiency values with BCC model
S1	1	1
S2	0.6442	1
S3	1	1
S4	0.583	0.5888
S5	0.5087	0.509
S6	0.7538	0.7978
S7	0.6953	0.6955
S8	1	1
S9	1	1
S10	0.8269	1
S11	1	1
S12	0.3242	0.3262
S13	0.1833	0.192
S14	0.3227	0.3328
S15	0.4194	0.4249
S16	0.3138	0.3149
S17	1	1
S18	0.9931	1
S19	0.5564	0.5659
S20	0.6142	0.6149
S21	0.3292	0.3383
S22	0.4081	0.4124
S23	0.8254	0.8917
S24	0.4942	0.495
S25	0.4699	0.5088

In table (6), according to CCR model results; the number of efficient units has increased compared with the results of Demir in table (2) and the unit 9 became efficient. While in the BCC model, the results are similar to table (2) so, nine units are efficient and the rest of the units are deemed inefficient relative to other efficient units.

Table (7) : Upper frontier efficiency scores with DEA models (CCR / BCC)

DMU	Upper efficiency values with CCR model	Upper efficiency values with BCC model
S1	0.4448	1
S2	0.1839	1
S3	0.8485	1
S4	0.2034	0.3957
S5	0.1609	0.2628
S6	0.2663	0.5601
S7	0.2846	0.3465
S8	0.5233	1
S9	0.34	1
S10	0.2988	1
S11	1	1
S12	0.0957	0.1041
S13	0.0503	0.0517
S14	0.0897	0.0982
S15	0.1145	0.139
S16	0.0923	0.0968
S17	1	1
S18	0.4471	1
S19	0.1926	0.1981
S20	0.1535	0.1606
S21	0.101	0.1099
S22	0.1256	0.1269
S23	0.37	0.5654
S24	0.1707	0.2165
S25	0.1746	0.2072

In table (7), according to CCR model results; it is found that the number of efficient units has decreased compared with the results in table (3) and the unit 3 turned to inefficient unit. While in the BCC model, the results are similar to table (3) so, nine units are efficient and the rest of the units are deemed inefficient relative to other efficient units.

Table (8) : Lower frontier efficiency scores with DEA models (CCR / BCC)

DMU	Lower efficiency values with CCR model	Lower efficiency values with BCC model
S1	1	1
S2	0.6702	1
S3	1	1
S4	0.5975	0.6085
S5	0.5345	0.5346
S6	0.7863	0.8144
S7	0.7192	0.7208
S8	1	1
S9	1	1
S10	0.8538	1
S11	1	1
S12	0.3544	0.3554
S13	0.196	0.2105
S14	0.3505	0.3661
S15	0.4523	0.4603
S16	0.3442	0.3476
S17	1	1
S18	1	1
S19	0.5883	0.6027
S20	0.6449	0.6485
S21	0.3472	0.3646
S22	0.4424	0.4501
S23	0.8511	0.9032
S24	0.5197	0.5227
S25	0.4888	0.5366

In table (8), according to CCR model results; the number of efficient units has increased compared with the results of Demir in table (2) and the units 9 and 18 became efficient. While in the BCC model, the results are similar to table (2), so, nine units are efficient and the rest of the units are deemed inefficient relative to other efficient units.

Table (9) : Upper frontier efficiency scores with DEA models (CCR / BCC)

DMU	Upper efficiency values with CCR model	Upper efficiency values with BCC model
S1	0.2543	1
S2	0.0985	1
S3	0.6297	1
S4	0.1069	0.3472
S5	0.0857	0.2147
S6	0.1479	0.5053
S7	0.1703	0.2815
S8	0.4214	1
S9	0.2276	1
S10	0.1859	1
S11	1	1
S12	0.0511	0.0617
S13	0.0251	0.0273
S14	0.0486	0.0582
S15	0.0638	0.0924
S16	0.049	0.0542
S17	1	1
S18	0.3164	1
S19	0.1141	0.1186
S20	0.0914	0.1047
S21	0.0522	0.062
S22	0.0698	0.0709
S23	0.2368	0.5133
S24	0.0913	0.1604
S25	0.0716	0.1219

Considering 99% confidence interval, efficiency values of interval DEA models (CCR / BCC) are calculated and placed on table (8) for lower frontier efficiency and table (9) for upper frontier efficiency as follows:

In table (9), according to CCR model results; it is found that the number of efficient units has decreased compared with the results in table (3) and the unit 3 turned to inefficient unit. While in the BCC model, the results are similar to table (3) so, nine units are efficient and the rest of the units are deemed inefficient relative to other efficient units.

IV. FINAL RESULTS AND CONCLUSION

The final results for the efficient units via classical DEA models and interval DEA models; the model proposed by Demir and the suggested approach using 90%, 95% and 99% confidence intervals are summarized in the following tables (10) the CCR model and (11) for the BCC model as follows:

Table (10): Efficient units using CCR model

Classical DEA	Lower and upper frontier efficient (Demir)		Lower and upper frontier efficient (90%)		Lower and upper frontier efficient (95%)		Lower and upper frontier efficient (99%)	
	L	U	L	U	L	U	L	U
S3 S8 S11 S17	S1 S3 S8 S11 S17	S3 S11 S17	S1 S3 S8 S9 S11 S17	S11 S17	S1 S3 S8 S9 S11 S17	S11 S17	S1 S3 S8 S9 S11 S17 S18	S11 S17

Table (11): Efficient units using BCC model

Classical	Lower and upper frontier efficient	Lower and upper frontier efficient	Lower and upper frontier	Lower and upper frontier efficient

DEA	(Demir)		(90%)		efficient (95%)		(99%)	
	L	U	L	U	L	U	L	U
S1	S1	S1	S1	S1	S1	S1	S1	S1
S2	S2	S2	S2	S2	S2	S2	S2	S2
S3	S3	S3	S3	S3	S3	S3	S3	S3
S8	S8	S8	S8	S8	S8	S8	S8	S8
S9	S9	S9	S9	S9	S9	S9	S9	S9
S10	S10	S10	S10	S10	S10	S10	S10	S10
S11	S11	S11	S11	S11	S11	S11	S11	S11
S17	S17	S17	S17	S17	S17	S17	S17	S17
S18	S18	S18	S18	S18	S18	S18	S18	S18

The results of the study are collected and shown in tables (10) and (11). Table (10) showed the results of efficient units in different cases. According to these results, units 11 and 17 remained efficient in all cases. Another point of view to the lower bound frontier efficient; if the confidence interval is increased; more units can be efficient compared to the results of Demir and classical DEA models. In other words, when the 90% and 95% confidence intervals were applied, the unit (9) became efficient although it is not efficient in the results of Demir and classical DEA models. Also, when the confidence interval became larger, namely 99%, another unit (18) lies on the efficiency frontier and becomes an efficient unit.

According to the results of BCC model in table (11), there is no difference between efficient units in all cases and all the results are the same; so nine units, namely, S1, S2, S3, S8, S9, S10, S11, S17, and S18 were identified as the best practice units.

REFERENCES

Abd-Aziz, N. A., Janor, R.M., Mahadi, R. (2013). Comparative departmental efficiency analysis within a University: a DEA approach. *Procedia - Social and Behavioral Sciences, vol. 90 pp.540 – 548.*

Argyrioy, G., and Sifaleras, A. (2013). An AMPL optimization software library for data envelopment analysis. *XL Balkan Conference on Operational Research*.

Avkiran, N.K. (2001). Investigating technical and scale efficiencies of Australian Universities through data envelopment analysis. *Socio-Economic Planning Sciences, Vol. 35, PP. 57-80.*

Banker, R., Charnes, A., and Cooper, W.W. (1984). Some models for estimating technical and scale inefficiencies in Data Envelopment Analysis. *Management Science, Vol. 30, pp. 1078-1092.*

Charnes, A., Cooper, W.W. & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operations Research, Vol. 2, pp. 429-444.*

Demir, E. (2014). A comparison of classical and fuzzy data envelopment analyses in measuring and evaluating school activities. *Turkish Journal of Fuzzy Systems, Vol. 5, No. 1, pp. 37-58.*

Farrell, M. (1957). The measurement of productive efficiency. *Journal of the Royal Statistical Society, Vol.120, pp.252-29*

Iddrisu, A. (2014). Measuring rural bank efficiency in Ghana: an application of data envelopment analysis (DEA) approach. *Journal of Human and Social Research, Vol. 3, No. 2, pp.41-59.*

Isabels, K.R., and Uthra, G. (2012). "An application of linguistic variables in assignment problem with fuzzy costs". *International Journal of Computational Engineering Research*, Vol. 2 Issue. 4.

Jablonsky, J. (2013). "Two-stage data envelopment analysis model with interval inputs and outputs ". *International Journal of Trade, Economics and Finance*, Vol. 4, No. 1, pp. 55-59.

Kazemi, M., and Alimi, A. (2014). "A fully fuzzy approach to data envelopment analysis". *Journal of Mathematics and Computer Science*, Vol. 11, pp. 238-245.

Mansourirad, E., Rizam, M.R.A., Lee, L.S., and Jaafar, A. (2010)." Fuzzy weights in data envelopment analysis". *International Mathematical Forum*, Vol. 5, No. 38, pp. 1871 – 1886.

Mohammadi, A., and Ranaei, H. (2011). "The application of DEA based Malmquist productivity index in organizational performance analysis". *International Research Journal of Finance and Economics*, ISSN 1450-2887 Issue 62.

Sengupta, J.K. (1992). "A fuzzy systems approach in data envelopment analysis". *Computers and Mathematics with Applications*, Vol. 24, pp. 259–266.

Tlig, H. (2013). "A fuzzy data envelopment analysis model to evaluate the Tunisian banks efficiency". *International Journal of Scientific & Engineering Research*, Vol. 4, Issue 9.

Wang, Y., Greatbanks, R. Yang, J. (2005). " Interval efficiency assessment using data envelopment analysis ". *Fuzzy Sets and Systems*, vol. 153, pp.347–370.

Zadeh, L. A. (1965). "Fuzzy Sets." *Information and Control*, Vol. 8, PP. 338-353.

Appendix (A)

DMU_S	INPUTS			OUTPUTS				
DMU	Number of teachers	Number of students	Number of classroom branches	YGS-LYS success rates	YGS point averages	LYS-score MS	LYS-score TM	LYS-score TS
S1	28	389	16	43.18	397.197	393.739	275.408	245.372
S2	45	509	26	60	351.484	296.973	288.684	203.346
S3	19	188	8	73.17	304.615	191.199	285.571	234.108
S4	46	559	21	48.48	318.434	268.463	256.699	185.058
S5	47	599	26	51.77	309.683	236.521	263.151	210.963
S6	33	288	16	59.62	285.805	228.089	265.58	175.905
S7	24	334	15	32.47	257.519	186.405	214.908	214.787
S8	13	310	11	47.17	271.478	178.161	234.892	187.212
S9	14	297	15	41.67	274.841	175.805	230.242	225.325
S10	22	310	16	56.06	282.584	197.133	233.308	238.603
S11	17	95	6	52.38	206.133	176.156	206.204	170.492
S12	53	930	32	21.93	224.725	160.665	205.977	203.182
S13	115	1272	51	20.45	213.992	170.764	196.905	185.845
S14	50	960	33	11.15	209.917	177.579	182.54	194.1
S15	38	650	28	15.66	213.286	182.659	185.22	205.618
S16	52	782	32	16.67	210.327	162.205	197.349	197.395
S17	9	121	7	31.43	204.448	165.59	191.659	209.788
S18	10	190	11	18.75	197.417	168.721	172.538	218.994
S19	24	355	18	20.79	208.82	158.187	189.875	200.079
S20	23	460	23	27	219.623	153.63	205.83	199.751
S21	58	770	28	11.48	196.202	176.865	182.087	190.464
S22	36	507	24	21.05	201.578	161.343	190.665	189.435
S23	19	188	12	17.02	180.082	171.72	173.274	222.271
S24	44	520	23	41	285.701	193.914	258.752	179.623
S25	96	475	17	43.06	229.314	137.388	201.315	193.933