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# Behavior of Goal Programming with Heavy Tailed Distribution

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**Abstract:** Goal programming (GP) is an extension of linear programming (LP). The GP is an important technique for decision makers. Goal programming technique has a useful advantage in minimize the unwanted deviations between the achievement of goals and their aspiration levels. The purpose of regression analysis is to expose the relationship between a response variable and predictor variables. In real applications, the response variable cannot be predicted exactly from the predictor variables. The response for a fixed value of each predictor variable is a random variable. For this reason, the behavior of the response may be summarized for fixed values of the predictors using measures of central tendency. Typical measures of central tendency are the average value (mean), the middle value (median) or the most likely value (mode). The main purpose for this study is to compare between two statistical method and one operation research method when these method used to estimate multiple linear regression equation with heavy tailed distribution. A simulation study based on four performance indexes to evaluate the performance of the three methods. The study suggested root mean square error with respect to the median (RMSEM) to use as a criteria to compare between three methods under consideration.The aim of this study is to study the behavior of goal programming and OLS, ALV to estimate the parameter of simple linear regression.

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## 1-INTRODUCTION

A very common and important problem in statistics is linear regression, the problem of fitting a straight line to statistical data. The most commonly employed technique is the method of least squares, but there are other interesting criteria where linear programming can be used to solve for the optimal values of the regression parameters.

The use of regression analysis depends on the choice of a criterion in order to estimate the coefficients of the explanatory variables. Traditionally, the ordinary least squares (OLS) as a statistical method however, Goal programming GP technique and Least Absolute value method as an operations research methods. LAV regression coefficients are chosen to minimize the sum of the absolute values of the residuals, by minimizing sums of absolute values rather than sums of squares. This paper introduced three efficiency indexes to examine the performance of OLS, GP and LAV coefficient estimators when the regression errors come from heavy tailed distribution such as Cauchy, Chi-square and skewed normal distributions.

The organization of the study is as follows: In Section 2 the study described ordinary least squares methods which used in this study. Section 3 described the linear goal programming applied in this simulation study. Section 4 described the least absolute values. Section 5 suggested the performance measures for efficiency methods. Simulation study discusses in Section 6. Finally, Results and discussion concluding remarks are provided in Section 7.

## 2- ORDINARY LEAST SQUARES

Ordinary least squares (OLS) is a mathematical method often used to numerically estimate a linear relationship between a continuous dependent variable and one or more independent or explanatory variables using sample data. The OLS estimator produces the best linear unbiased estimate of the relationship between each independent/explanatory variable and a continuous dependent variable while simultaneously eliminating the linear effects of the other included independent variables. The principle of OLS was first published by A. Legendre in 1805. Even today OLS remains a popular analytical tool in the analysis of social data.

The least squares methods are computational technique for determining the best equation describing set points where best is defined geometrically [8]

Suppose that the dependent variable Y can be expressed as a linear function of n predictor variables $X_1, X_2, ..., X_n$:

$$Y = \beta_0 + \beta_1 X_1 + ... + \beta_n X_n + \varepsilon_i \quad (1)$$

Where $\beta = [\beta_0 \, \beta_1 \ldots \beta_n]'$

is the vector of regression coefficient to be estimated, each observation ( $x_{ij}, \ldots, x_{in}, y_i$ ) satisfies the equation:

$$y_i = \beta_0 + \beta_i \, x_{i1} + \ldots + \beta_n \, x_{in} + \varepsilon_i$$
$$\hat{y}_i = b_0 + b_1 \, x_{i1} + \ldots + b_n \, x_{in} + e_i \qquad (2)$$

The least squares estimates ($L_2$) of regression coefficients are the values $\beta_0$ , $\hat{\beta}_i$ obtained by

$$L = \sum_{i=1}^{n} e_i^2 \qquad (3)$$

**3- Linear Goal Programming**

Goal Programming (GP) is the most widely used approach in the field of multiple criteria decision making that enables the decision maker to incorporate numerous variations of constraints and goals. Linear GP problems are GP problems where each objective function is linear. It was first developed and introduced by A. Charnes and W.W Cooper in 1961 and further refined by Y. Ijiri in 1965. According to Charnes and Cooper [1], GP extends the linear programming formulation to accommodate mathematical programming with multiple objectives. [3]

GP's objective function is always minimized and must be composed of deviational variables only.. In the formulation, two types of variables are used: decision variables and deviational variables. There are two categories of constraints: structure/ system constrains and goal constrain, which are expressions of the original functions with target goals set a priori and positive and negative deviational variables[9].

The general GP model can be expressed as follows

$$\left.\begin{aligned}
&Minimize \;\; Z = \sum_{i=1}^{m} \; (d_i^- + d_i^+) \\
&Subject\ to\ linear\ constrians: \\
&Goal\ constrain: (\sum_{i=1}^{n} a_{ij} \, x_j) + d_i^- - d_i^+ = b_i \,, \quad i = 1,2,\ldots,m \\
&system\ constrain: \;\; \sum_{i=1}^{n} a_{ij} \, x_j \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b_i \;\; , i = m+1, \ldots., m+p \\
&with \;\; x_j \;\; , d_i^- \, , d_i^+ \geq 0 \; , \, for \;\; i = 1, 2, \ldots, m \;\; and \;\; j = 1, 2, \ldots, n.
\end{aligned}\right\} \qquad (4)$$

**4- LEAST ABSOLUTE VALUE**

Least absolute value (LAV) regression methods have been widely applied in estimating regression equations. However, most of the current LAV methods are based on the original goal program developed over four decades. Since Charnes et al. [2] formulated the least absolute value (LAV) regression problems as linear programming. Numerous algorithms have been developed to solve LAV regression problems. Dodge [5] and Dielman [4] thoroughly review these algorithms.

LAV regression algorithms based on the conventional goal programming techniques as follows:

$$\left.\begin{aligned}
&Minimize \sum_{i=1}^{n} \; (d_i^+ + \, d_i^+) \\
&subject\ to \;\; y_i - (b_0 + \sum_{j=1}^{m} x_{ij} b_j + d_i^+ - d_i^-) = 0, i = 1,2,\ldots,n,
\end{aligned}\right\} \qquad (5)$$

Where $d_i^+$ and $d_i^-$ is, respectively, the positive and negative deviation variable associated with the $i$ the observation and $d_i^+ \, , d_i^- \geq 0 \;\; and \; b_j \; (j = 0,1,2,\ldots,m) \in F \; (F\ is\ a\ \ feasible\ set)$ [6].

**5- The Performance Measures**

In this study four performance measures (mean square errors, G, $R^2$, and root mean square error with respect to the median) are used as criteria to compare between the ordinary least squares, Goal programming and least absolute value estimators.

$$ER(\hat{\alpha}) = MSE = \sum_{r=1}^{R} (\hat{\alpha} - \alpha)^2 / R \qquad (6)$$

$$ER(\hat{\beta}) = MSE = \sum_{r=1}^{R} (\hat{\beta} - \beta)^2 / R \qquad (7)$$

Where *R* is the number of repeated samples and α, β are regression coefficient.(Ismail 2003)

The coefficient of determination $R^2$ is popular statistic in the analysis of regression models. An intuitively appealing alternative to least squares in such situations is the least absolute deviations (LAV) principle. The LAV fit of a linear model is known to be resistant to the influence of highly efficient for heavy tailed distributions. It would be useful then to have an analog of the classical $R^2$ for the LAV analysis of a linear model as follow:

$$R^2 = 1 - \frac{Z}{\sum (y_i - \bar{y})^2} \qquad (8)$$

Z is the value of objective function for three methods.

- Classical regression models are obtained by choosing a model that minimizes an empirical estimation of the Mean Square Error (MSE). Another quality measure of regression is given by the Mean Absolute Percentage Error (MAPE) (G). If x denotes the explanatory variables (the input to the regression model), y denotes the target variable, the MAPE is obtained by:

$$G = 1- [Z / \sum_{r=1}^{R} | \ (y_i - median) \ |] \quad , \qquad (9)$$

Where $y_i$are the values dependent variables.

The MAPE is often used in practice because of its very intuitive interpretation in terms of relative error. A large value of G means that the value of the error is large. The use of the MAPE is relevant in finance, for instance, as gains and losses are often measured in relative values. It is also useful to calibrate prices of products, since customers are sometimes more sensitive to relative variations than to absolute variations.

- In connection with the MSE, another index, namely the root mean square error with respect to the mean (RMSEM) is also by: [3]

$$ER(\hat{y}) = MSE = \sum_{r=1}^{R} (\hat{y} - y)^2 / R \qquad (10)$$

The suggested index which proposed in this paper is ($RMSEM_d$) defined as follows:

$$RMSEM_d = (MSR)^{1/2} / \overline{median} \qquad (11)$$

**6-SIMULATION STUDY**

This section discusses the numerical simulation of the three models, ordinary least squares, Goal programming and least absolute value. Three methods under consideration are used to estimate the parameters of the simple linear regression equation. The study applied error distribution has heavy tailed,Cauchy, Chi-square and skewed normal distributions. The steps which taken place in this simulation study as follows:

- The regression model was used as $y = x_i' \beta + \varepsilon_i$ or ( y=5 + 3x + $\varepsilon_i$ ).

- Where study generated $\varepsilon_i$ from three distributions with two parameters.

- Cauchy, Chi-square and skewed normal distributions were employed to generate a heavy tailed distribution to estimate the parameters of the regression model.

-Random samples of size n=10, 15 and 20 are generated.
This study introduced a program by using GAMS 2.25 statistical package to calculate ordinary least squares,least absolute value and Goal programming.

-Three methods which used to estimate regression equation with error distribution and follow each of the following three distributions and their parameters respectively,
Cauchy ~ C (0, 1.1) and (0, 3);
Chi-square ~ $\chi 2_{(4)}$ and $\chi 2_{(6)}$;
Skewed normal ~ N (0, 15) and (0, 25).

-These distributions have been generated using the above parameters that were chosen arbitrarily and taken from many previous studies to calculate performance the methods under considerations.

-The study applied sampling runs (number of repeated) 500 replications for each distribution $i^{th}$ the two different parameters and three different sample sizes to be sure of consistency of the results.

-For all sample sizes, for all methods, and for all distribution parameters for the three distributions, the MSE,R,G and $RMSEM_d$ for each parameters were calculated using each method separately.

-The criteria to evaluate the performance for the three methods under considerations depend on the approach, which produce a small MSE and small $RMSEM_d$ , for all parameters then it would be considered more suitable when the objective is to select the variables and estimate the parameters.

-Table (1) to Table (3) showed all performance measures; MSR, $R^2$, G and $RMSEM_d$ for the three methods, ordinary least squares, Goal programming and least absolute value.

**Table (1): The three methods with Cauchy (0, 1.1) and(0.3) with different sample size.**

| | | N=10 | | | N=15 | | | N=20 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | OLS | GP | LAV | OLS | GP | LAV | OLS | GP | LAV |
| $\rho$ =1.1 | MSE (a) | 1.420068E+9 | 348.624 | 9.048572E+7 | 3.746918E+9 | 1329.889 | 1858098.695 | 234447.616 | 153.789 | 1.054254E+7 |
| | MSE (b) | 1.420055E+7 | 34.441 | 1.003190E+7 | 1.665608E+7 | 16.799 | 1127358.661 | 961.097 | 2.078 | 2512472.909 |
| | $R^2$ | 0.118 | 1.000 | 0.560 | 0.026 | 1.000 | 0.933 | -0.326 | 1.000 | 0.949 |
| | G | -1.89137E+7 | 1.000 | -6.477 | -6.59248E+7 | 1.000 | -5.040 | -221.561 | 1.000 | -0.741 |
| | $RMSEM_d$ | 7494.759 | 492.547 | 492.547 | 22220.356 | 724.131 | 724.131 | 1127.223 | 906.728 | 906.728 |
| $\rho$ =3 | MSE (a) | 1.05625E+10 | 2593.070 | 6.730343E+8 | 2.78692E+10 | 9891.733 | 1.382057E+7 | 2346.502 | 1143.887 | 7.841560E+7 |
| | MSE (b) | 1.056240E+8 | 256.171 | 7.461747E+7 | 1.238860E+8 | 124.949 | 8385312.352 | 12.506 | 15.459 | 1.868782E+7 |
| | $R^2$ | -1.751 | 1.000 | 0.420 | -0.889 | 1.000 | 0.931 | -423508.872 | 1.000 | 0.892 |
| | G | -3.75066E+7 | 1.000 | -20.916 | -4.30509E+7 | 1.000 | -4.477 | -4.15658E+7 | 1.000 | -3.184 |
| | $RMSEM_d$ | 12166.435 | 799.564 | 799.564 | 36718.934 | 1196.625 | 1196.625 | 1435.435 | 1492.097 | 1492.097 |

**Cauchy distribution:** From the results in Table (1) three different sample sizes (10,15, 20) and two different parameters (0, 1.1), (0, 3) for Cauchy distribution using OLS, G.P and LAV methods. This study observed that the results of MSE for (a) and (b) is large for or all sample sizes with two parameters, when, $R^2$ and G efficiency with goal programming. $RMSEM_d$is almost the same with samples 15 and 20.

**Table (2): The three methods with Chi-square ~ $\chi2_{(4)}$and $\chi2_{(6)}$with different sample size.**

| | | N=10 | | | N=15 | | | N=20 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | OLS | GP | LAV | OLS | GP | LAV | OLS | GP | LAV |
| $\rho$ = 4 | MSE (a) | 1.447913E+9 | 348.621 | 9.048572E+7 | 3.765560E+9 | 1329.854 | 1855353.206 | 234447.616 | 153.789 | 1.054254E+7 |
| | MSE (b) | 1.447900E+7 | 34.440 | 1.003190E+7 | 1.673895E+7 | 16.799 | 1133115.535 | 961.097 | 2.078 | 2512472.909 |
| | $R^2$ | 0.117 | 1.000 | 0.559 | 0.025 | 1.000 | 0.933 | -0.326 | 1.000 | 0.949 |
| | G | -1.90410E+7 | 1.000 | -6.476 | -6.59907E+7 | 1.000 | -5.063 | -221.561 | 1.000 | -0.741 |
| | $RMSEM_d$ | 7564.695 | 491.955 | 491.955 | 22288.825 | 724.574 | 724.574 | 1127.223 | 906.728 | 906.728 |
| $\rho$ = 6 | MSE (a) | 1.219634E+9 | 288.122 | 7.478169E+7 | 3.127439E+9 | 1099.054 | 1527527.952 | 193755.731 | 127.100 | 8670337.718 |
| | MSE (b) | 1.219623E+7 | 28.478 | 8290831.548 | 1.390233E+7 | 13.883 | 931634.341 | 794.296 | 1.718 | 2076301.144 |
| | $R^2$ | 0.178 | 1.000 | -3.252 | -0.141 | 1.000 | 0.941 | -0.212 | 1.000 | 0.955 |
| | G | -1.90788E+7 | 1.000 | -9.655 | -6.22673E+7 | 1.000 | -4.742 | -188.703 | 1.000 | -0.623 |
| | $RMSEM_d$ | 7279.888 | 468.902 | 468.902 | 21311.435 | 687.055 | 687.055 | 1060.202 | 858.984 | 858.984 |

**Chi-square distribution:** From the results in Table (1) three different sample sizes (10,15, 20) and two different Degrees of freedom (4), (6) for Cauchy distribution using OLS, G.P and LAV methods. This study observed that the results of MSE for (a) and (b) is large for or all sample sizes with two parameters, when, $R^2$ and G efficiency with goal programming. $RMSEM_d$is almost the same with samples 15 and 20.

**Table (3): The three methods with Skewed normal ~ N (0, 15) and Skewed normal ~ N (0,25) with different sample size.**

| | | N=10 | | | N=15 | | | N=20 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | OLS | GP | LAV | OLS | GP | LAV | OLS | GP | LAV |
| $\rho$ =15 | MSE (a) | 91.947 | 286.905 | 16248.366 | 67.042 | 235.122 | 8839.081 | 54.313 | 294.876 | 8232.368 |
| | MSE (b) | 2.604 | 5.780 | 428.115 | 0.854 | 2.046 | 360.796 | 0.370 | 1.484 | 330.893 |
| | $R^2$ | -1.812 | 1.000 | -1.056 | -1.847 | 1.000 | 0.920 | 0.266 | 1.000 | 0.955 |
| | G | -121.770 | 1.000 | 26.640 | -80.352 | 1.000 | -4.356 | -67.575 | 1.000 | -1.912 |
| | $RMSEM_d$ | 214.392 | 279.952 | 279.952 | 368.945 | 525.403 | 525.403 | 488.553 | 744.916 | 744.916 |
| $\rho$ =25 | MSE (a) | 255.408 | 796.957 | 45134.350 | 186.228 | 653.118 | 24553.003 | 150.869 | 819.099 | 22867.688 |
| | MSE (b) | 7.234 | 16.054 | 1189.207 | 2.371 | 5.683 | 1002.211 | 1.028 | 4.121 | 919.148 |
| | $R^2$ | -16.110 | 1.000 | 0.395 | -4.342 | 1.000 | 0.958 | -4.559 | 1.000 | 0.940 |
| | G | -399.361 | 1.000 | -91.173 | -228.301 | 1.000 | -6.722 | -131.077 | 1.000 | -3.142 |
| | $RMSEM_d$ | 261.707 | 341.736 | 341.736 | 477.198 | 679.563 | 679.563 | 620.266 | 945.744 | 945.744 |

**Skewed normal distribution:** From the results in Table (3) three different sample sizes (10,15, 20) and two different parameters (0,15), (0,25) for Cauchy distribution using OLS, G.P and LAV methods. This study observed that the results of MSE for (a) and (b) is large for or all sample sizes with two parameters, when, $R^2$ and G efficiency with goal programming. $RMSEM_d$is almost the same with samples 15 and 20.

## 7. CONCLUSION

This section concerned with the results related with simulation study for three methods under consideration; the three methods of linear programming ordinary least square, Goal programming and least absolute value. The three methods are used to estimate the parameters of the regression equation when three distributions with three different sample sizes and two different parameters for each error distribution. Goal programming for estimation of parameters when Cauchy distribution, Chi-square and Skewed normal are used much better than the OLS and LAV approaches.

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