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Examination of the relationship between generating examples and checking examples in children

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Abstract: Examples play an important role in teaching mathematics. Thus, the present research aims at study of the relationship between examples and understanding mathematics. To do so, the following questions are raised: when generating and checking examples, one aspects is given attention or different aspects? Does the learner's ability in generating more examples make the learner not to check the examples?. To collect the data, the subjects were interviewed with all elementary students in grade three, four and five. were selected that through interview and tracking students' intellectual method via frequent questions and controlling their drafts, their generated examples become identified. It should be mentioned names are unreal. The results show that there are differences between "generating example" and "checking examples". Also, it looks that a strong relationship is available between generating example for a concept and understanding that concept.

RESEARCH BACKGROUND

According to Housman and Dahlberg (1997), individuals generate examples through the following methods:
-using memory
-using trial and error method
-examples are generated with reviewing answers of previous problems (trial and corrected error method).

This classification categorizes examples based on the way peoples generate examples. However, Mikner (1978) classifies examples only according to nature of the example and not the individuals. They are as follows:
-startup examples
-reference examples
-generic examples
-counter examples

Although the above mentioned studies pay attention to study of examples, several other researches use examples as a means for evaluation of their desired concepts. Vinner and Tall(1981) through study of examples were checked by students, tracked their intellectual line to show that for study of the concept of continuity adopted concept image instead of concept definition. Also, Mason and Watson(2005) stated that there is a substantial difference among providing example by the student and generating that example by other students.

In literature, generating examples and checking examples are two independent operations. But Akbari (2007) pointed that generating examples and checking examples refer to different aspects of the same concept. Also, he said that the way an example is generated affects on checking or not checking of examples. The present research, therefore, is an attempt to find a clearer relationship between generating example and checking example.

METHODOLOGY

To study the relationship between generating example and checking example, elementary students of grade three, four and five were invited to interview. The interview session was 30-40 minutes. Thanks to particular conditions of interviewees (children aged 9-11), singular and face to face interviews could put unwanted pressure on students and decrease the student's efficiency. Thus, interviews were held in 2-3 people session. To avoid stress in interview, before doing research the interview created a friendly and stress free environment with students.

The following questions were asked and their answers were evaluated. Before analyzing the interview, let's take a look at the questions:
Problem 1: product of two numbers is 72, what are those numbers?

By this question, the researcher aimed to find paired numbers like (72,1), (8,9) ,(6,12),etc. the reason for choosing 72 is large denominators of this number helped students to fined examples. Many students by use of their memory could find (8,9) pair though they were weak in finding other pairs.

Due to difficulty of many students in multiplication, after being used in the few primary interviews it was put aside.

Problem 2: sum of three figures is 123. What are those numbers?
By this question, the purpose was finding numbers their sum is 123. Using this number was intentional. Students usually are more familiar with numbers like 2, 5,10,50,100,and 1000 rather than numbers 93,38,or 123. Since they have realized numbers like 10,100 and 1000 as tens, hundreds and thousands inside math books as well as existing bills in the society so, they were aware of some characteristics of these figures beforehand. Consequently, numbers 123 were selected

This choice helped instead of paying attention to the number itself, students concentrated on the problem, however, in some occasions based on research needs , number 100 (for more easiness) replaced with 123.
Three pairs (23,50 and 50) were among the most common answers students gave to this question. In fact, in spite of not selecting number 100, the students' familiarity with it affected the problem. Through using statements like asking to generate three different examples could manage the interview efficiently in order to stimulate students to find other alternatives.

Problem 3: say three numbers their sum equals to 100.
For those students who acted unsuccessfully in the previous problem, the current question was a suitable problem with less operations and computations. Most of students initiated from two numbers 50+50 and gradually found more solutions to the problem.

Problem 4: write two numbers their difference equals to 47.
Like problem 3, the number 47 was selected because of less familiarity of students. However, this time instead of using a three digit number, a 2 digit number was selected. Unlike its simple statement, most of students had difficulty in finding the first answer. Perhaps it was due to knowing less about subtraction compared to addition.

Analyzing the answers:
Considering the type o answer students gave and cracking their intellectual line by certain common questions besides paying attention to their drafts were the research procedures. To answer the research question it was necessary that evaluate the nature and process of generating examples within methods were explained by Dahlborg and Housman (1997). That is the process of test and correct error plus the process of generating example by use of generative algorithm.

Mohsen (third grade)answers the question: " sum of three numbers equals to 100, what are those numbers?" as follows:
I: Mohsen, sum of three numbers equals to 100. What are the numbers?
Mohsen:

$$\begin{array}{r} 70 \\ +20 \\ \underline{+10} \\ 100 \end{array}$$

At first I didn't realized whether he used his memory or not. But as I continued it became clearer.
I: can you say some other examples?
Mohsen: yes, (here, he first wrote 50 and 15 immediately and 35 with a delay).

$$\begin{array}{r} 50 \\ +15 \\ \underline{+35} \\ 100 \end{array}$$

I: good job, and another example:
Mohsen:

$$\begin{array}{r} 23 \\ +36 \\ \underline{+41} \\ 100 \end{array} \quad , \quad \begin{array}{r} 40 \\ +12 \\ \underline{+48} \\ 100 \end{array}$$

Like the previous cases, he wrote 48 and 41 with more delay compared to first two numbers. In fact, the first two numbers were randomly selected but the third number was calculated through the following mental computation:
100-(36+23)=41 100-(40+12)=48

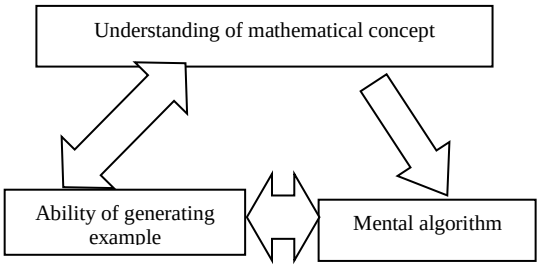
The next answers were also interesting:
66, 18, 16), (50 ,49, 1),(86 ,8 ,6) (60, 11, 29). . . (.

As it can be seen, there is no special association between Mohsen's answers. The only common thing is that the first two numbers were randomly selected though the third number was computed by adding two first numbers and subtracting from 100.

The important point in this process is the student's awareness of properties of addition and subtraction. That is, if Mohesn did not know addition and subtraction are related, he could not use such method.

Therefore, awareness and true understanding from mathematical concepts enabled Mohsen to choose a proper strategy for solving the problem,. This issue is confirmed about peoples use generative algorithm for generating examples either. As a matter of fact, other participants adopted "generative algorithm" and "corrected trial and

error" were familiar with such a concept. Considering these instances it can be claimed that generating example strongly link to perception and understanding of mathematical concepts. As generating example for a concept contributes its better understanding, having a suitable conceptual understanding can improve ability of generating example in individuals. The following figures show these relationships.



The next question is that whether at time of generating and checking example , students are engaged with a series of concepts or concepts are used in generating example can differ from concepts are used in checking the same example. To know this, two students were evaluated.

Rasoul, the fourth grade student answers the problem" difference between two number equals to 47. What are the numbers?"
He used the interesting algorithm. That is, he added a constant value to 47 then considered the new number with the primary value as the answer. Below we can see answers are given by Rasoul:
47+7=54
54-7=47
47+11=58
57-11=47
Rasoul did not check his answers but he was also uncertain about their correctness. I asked Rasoul to check his answers to get sure.

Here, we can see the way Rasoul checked his answers.

$$\begin{array}{rr} 54 & 58 \\ -\,7 & -11 \\ \hline 47 & 47 \end{array}$$

To generate example, Rasoul used addition. However, for checking that example he subtracted. This means that for generating and checking example, the student was struggling with various concepts mentally. Hamed , a fourth grade student shows this issue in his work as well.

$$:123 - 77 = 46 \qquad 60 + 17 = 77$$

To find the sum of three number equals to 123, Hamed used addition and subtraction simultaneously. That is, first he selected two random values in his mind, then to get the third number, he subtracted the sum of first tow number from 123. For instance, he first considered two numbers 60 and 17 and computed the third number like this:

$$60 + 17 = 77$$
$$123 - 77 = 46$$

Now, Hamed checked each of his generated examples once more to ensure they are correct. In order to check examples, Hamed only used addition. For instance, to check three numbers, 46, 17,60 he wrote:

$$\begin{array}{r} 60 \\ +17 \\ \underline{+46} \\ 123 \end{array}$$

He also used either addition or subtraction for generating examples, though for checking his examples, he applied subtraction. This shows that at time of checking examples it is unnecessary to utilize exactly the same concepts are used in generating examples.

Considering the above mentioned instances plus other examples, we may assume that generating examples and checking examples are two independent processes. Concepts are used in each of these processes can or cannot be applicable in other process. Also, in some cases, the student knew that how generate an example but he didn't know how to check it. This especially occurs more about subjects generate mental examples.

As the research goes ahead, the researcher observed that students used generative algorithms for generating examples acted much better than other students in generating examples. If their algorithm was mathematically correct, the generated answer was true as well. But it looks that some students were unaware of it. Since they were uncertain about their generated examples and to get sure about their answer, they checked it once.

Through using a generative algorithm, Shayan found answers for the problem" finding three numbers equal to 123" . There were some significant points. Let's review the answers first.

$$\begin{array}{rrrrr} 80 & 70 & 90 & 60 & 70 \\ +20 & +30 & +\,3 & +3 & +13 \\ \underline{+23} & \underline{+23} & \underline{+30} & \underline{+60} & \underline{+40} \\ 123 & 123 & 123 & 123 & 123 \end{array}$$

Although Shayan knew that all above sets of numbers should be equal to 123, he calculated the one by one. The small 1(ten on one) shows his computation wrote on his drafts. To be sure where or not he knows the answers were achieved by algorithm, I continued the procedure initiated by

Shayan and wrote an answer for Shayan. Shayan had previously generated some answers.

60	50	40
+3	+3	+3
+60	+70	+80
123	123	123

I guess the next answer should be 30+3+90. So I wrote the answer in a linear format:
I: Shayan the next answer may be 30+3+90.
Shayan: (it seem he is checking the product to see whether it compatibles with the algorithm or not).
He said "Yes, sir, that's it".

I : so 30+3+90 equals to 123.
Shayan: sir? (He immediately wrote).

30
+3
+90
123

Yes sir, 30+3+90=123

Shayan use from a mental algorithm consciously in order to measure first assess my answer then confirm its correctness. Also, at the end of interview I asked Shayan to explain his procedure and he did it with his own words. However, he was not satisfied with answers of the algorithm until he checked their correctness with addition. For every single answer, Shayan checked them until the end.
Despite Shayan that had a good mastery on addition and subtraction as well as conscious use of generative algorithm, and checked all of his answers, Sina did differently. To answer the same problem, Sina also used a generative algorithm. Let's see his interview.

50
+50
+23
123

I : can you say three numbers their total equals to 123?
Sina: yes

50
+50
+23
123

I : can you say some more examples?
Sina: yes

60	70
+40	+30
+23	+23
123	123

Sina didn't check any of the above answer. He only wrote answers on the sheet.
I: now, can you give me an example?
Sina: yes

49	48
+50	+50
+24	+25
123	123

Sina wrote the answers soon.
I: are you sure they are correct? Don't you want to check them?
Sina: very confident, no sir, they must be correct.
I: can you give me more examples?

Here, something interesting happened. Sina's answers were incorrect.
Sina:

47
+50
+27
123

I: Sina are sure your answer is true?
Sina: yes sir, it must be true.
I: it would be better if you check the answer. Add the three numbers and see what the answer is?
Sina: Sir, the answer is 123. I don't need to add them.

In fact, to find the last answer, Sina committed a big mistake. Though he was persistent and did not check the answer any more. In following, I asked a few more question from Sina and again he didn't check any answers either correct or incorrect answers.

Taking all together, it can be concluded that after generating examples, checking or not checking it depends on something more than degree of mathematical understanding. In fact, degree of mathematical perception should not be taken as a reason for checking or not checking an example. Perhaps, factors affect on checking examples are psychological.

CONCLUSION:

Considering the results, it seems that there are differences between "generating example" and "checking examples". These difference exist in concepts are involved in these processes. For instance, though in the process of generating examples addition and subtraction are applied; it is likelihood for checking that example only addition is used.

Furthermore, it looks that a strong relationship is available between generating example for a concept and understanding that concept. This relationship could be mutual in a way that progress in one will lead to development in the other. This is evident for individuals who generate examples through "corrected trial and error" and "generative algorithm" methods. The last but not least, the

individuals' ability in generating example cannot be reason for checking or not checking the same example.

REFERENCES:

Asghari.AmirHossein,Examples, A missing link, PME, Vol 2,p25-32,2007

Dahlberg.Randall P, Housman.David.L, Facilitating learning evevnts through example generation, Educational studies in mathematics, 33 p283-289, 1997

Risslan Edwina & Michener, Understanding understanding mathematics, Cognitive science , 2, p361-383, 1978

Tall David, VinnerShlomo, Concept image and concept definition in mathematics with particular reference to limit and continuity, Educational studies in mathematics, 12, p151-169, 1981